Unit Step Function:-

With the help of unit step functions, we can find the inverse transform of functions, which cannot be determined with previous methods.

The unit step function $u(t-a)$ is defined as follows:

$$u(t-a) = \begin{cases} 0, & \text{when } t < a \\ 1, & \text{when } t \geq a \end{cases}$$

Then Laplace transform of unit step function is given by,

$$L\{u(t-a)\} = \int_0^\infty e^{-st}\, u(t-a)\, dt$$

$$= \int_0^a e^{-st}\, u(t-a)\, dt + \int_a^\infty e^{-st}\, u(t-a)\, dt$$

$$= \int_0^a e^{-st} \cdot 0\, dt + \int_a^\infty e^{-st}\, dt$$

$$= 0 + \left[\frac{e^{-st}}{-s}\right]_a^\infty$$

$$\boxed{L\{u(t-a)\} = \frac{e^{-sa}}{s}}$$

Ex:- Express the following function in terms of unit step functions and find its Laplace transform.

$$f(t) = \begin{cases} 8, & t < 2 \\ 6, & t \geq 2 \end{cases}$$

Soln:- $f(t) = \begin{cases} 8+0, & t<2 \\ 8-2, & t \geq 2 \end{cases} = 8 + \begin{cases} 0, & t<2 \\ -2, & t \geq 2 \end{cases}$

$$= 8 + (-2)\begin{cases} 0, & t<2 \\ 1, & t \geq 2 \end{cases}$$

$$= 8 - 2u(t-2)$$

$$\therefore L\{f(t)\} = 8L(1) - 2u(t-2)$$

$$= \frac{8}{s} - 2\frac{e^{-2s}}{s}$$

Scanned by CamScanner

Second shifting theorem

If $L[f(t)] = F(s)$ then $L[f(t-a)\cdot u(t-a)] = e^{-as}F(s)$

Proof:- $L[f(t-a)\cdot u(t-a)] = \int_0^\infty e^{-st}[f(t-a)\,u(t-a)]\,dt$

$= \int_0^a e^{-st} f(t-a)\cdot 0\,dt + \int_a^\infty e^{-st} f(t-a)(1)\,dt$

$= \int_a^\infty e^{-st} f(t-a)\,dt$

$= \int_0^\infty e^{-s(u+a)} f(u)\,du$ where, $u = t-a$

$= e^{-sa}\int_0^\infty e^{-su} f(u)\,du$

$= e^{-sa} F(s)$

Ex:- Express the following function in terms of unit step function and find its Laplace transform:

$$f(t) = \begin{cases} E, & a<t<b \\ 0, & t \geq b \end{cases}$$

Soln:- $f(t) = E\begin{cases} 1, & a<t<b \\ 0, & t \geq b \end{cases}$ $\left[L[f(t-a)\cdot u(t-a)] = e^{-as}F(s)\right]$

$\Rightarrow L\{f(t)\} = E\left[\frac{e^{-as}}{s} - \frac{e^{-bs}}{s}\right]$

Ex.2:- Express the following function in terms of unit step function:

$$f(t) = \begin{cases} t-1, & 1<t<2 \\ 3-t, & 2<t<3 \end{cases}$$

and find its Laplace transform.

Soln:- $f(t) = \begin{cases} t-1, & 1<t<2 \\ 3-t, & 2<t<3 \end{cases}$

$= (t-1)[u(t-1) - u(t-2)] + (3-t)[u(t-2) - u(t-3)]$

Scanned by CamScanner

$(t-1)u(t-1) - (t-1)u(t-2) + (3-t)u(t-2) + (t-3)u(t-3)$

$= (t-1)u(t-1) - 2(t-2)u(t-2) + (t-3)u(t-3)$

$= e^{-s} L(t) - 2e^{-2s} L(t) + e^{-3s} L(t)$

$= \frac{e^{-s}}{s^2} - 2\frac{e^{-2s}}{s^2} + \frac{e^{-3s}}{s^2}$

Q. 1) $F(t) = \frac{e^{-at} - e^{-bt}}{t}$ 2) $F(t) = \frac{\cos at - \cos bt}{t}$

3) $F(t) = \frac{\sinh t}{t}$ 4) $F(t) = \frac{\sin^2 t}{t}$

5) $F(t) = \frac{e^{-t} - e^{-3t}}{t}$

2) $\frac{1}{2} \log \frac{s^2+b^2}{s^2+a^2}$ 3) $-\frac{1}{2} \log \frac{s-1}{s+1} = \frac{1}{2} \log \frac{s+1}{s-1}$

4) $\frac{1}{4} \log \frac{s^2+4}{s^2}$ 5) $\frac{1}{2} \log \frac{s^2+b^2}{(s-a)^2}$

Scanned by CamScanner

Periodic Functions: (Laplace transform of periodic functions)

Let's consider a periodic function $F(t)$ of period T; so that $F(t+T) = F(t)$; $T>0$. Then the Laplace transform of $F(t)$ is given by,

$$L\{F(t)\} = \int_0^\infty e^{-st} F(t)\, dt$$

$$= \int_0^T e^{-st} F(t)\, dt + \int_T^{2T} e^{-st} F(t)\, dt + \cdots + \int_{nT}^{(n+1)T} e^{-st} F(t)\, dt + \cdots$$

$$= \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} e^{-st} F(t)\, dt$$

Substituting $t = x + nT$ i.e. $dt = dx$; we get

$$L\{F(t)\} = \sum_{n=0}^{\infty} \int_0^T e^{-s(x+nT)} F(x+nT)\, dx$$

$$= \sum_{n=0}^{\infty} \int_0^T e^{-sx} e^{-nsT} F(x)\, dx \quad [\text{since } F(x+nT) = F(x) \text{ for } n = 0, 1, 2, 3 \cdots]$$

$$= \sum_{n=0}^{\infty} e^{-nsT} \int_0^T e^{-sx} F(x)\, dx$$

$$= \frac{1}{1-e^{-sT}} \int_0^T e^{-sx} F(x)\, dx \quad \left[\text{since } 1 + e^{-sT} + e^{-2sT} + \cdots = \frac{1}{1-e^{-sT}}\right]$$

Thus,

$$\boxed{L\{F(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} F(t)\, dt}$$

OR

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^T e^{-st} f(t)\, dt + \int_T^{2T} e^{-st} f(t)\, dt + \int_{2T}^{3T} e^{-st} f(t)\, dt + \cdots$$

$$= \int_0^T e^{-st}$$

Substituting $t = u + T$ in second integral and $t = u + 2T$ in 3rd integral and so on...

Scanned by CamScanner

$$L\{f(t)\} = \int_0^T e^{-st} f(t)\,dt + \int_0^T e^{-s(u+T)} f(u+T)\,du$$

$$+ \int_0^T e^{-s(u+2T)} f(u+2T)\,du + \cdots$$

$$= \int_0^T e^{-st} f(t)\,dt + e^{-sT}\int_0^T e^{-su} f(u)\,du + e^{-2sT}\int_0^T e^{-su} f(u)\,du + \cdots$$

$$[f(u) = f(u+T) = f(u+2T) = f(u+3T) = \cdots]$$

$$= \int_0^T e^{-st} f(t)\,dt + e^{-sT}\int_0^T e^{-st} f(t)\,dt + e^{-2sT}\int_0^T e^{-st} f(t)\,dt + \cdots$$

$$= \int_0^T e^{-st} f(t)\,dt \left[1 + e^{-sT} + e^{-2sT} + e^{-3sT} + \cdots\right]$$

$$= \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$$

Ex.1 :- Find the Laplace transform of the waveform

$$f(t) = \left(\frac{2t}{3}\right), \quad 0 \le t \le 3$$

Sol^n :- $L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$ $\left[f(t) = \frac{2t}{3} \text{ with period } T = T = 3\right]$

$$L\left(\frac{2t}{3}\right) = \frac{1}{1-e^{-3s}} \int_0^3 e^{-st} \left(\frac{2t}{3}\right) dt$$

$$= \frac{1}{1-e^{-3s}} \cdot \frac{2}{3} \left[\frac{te^{-st}}{-s} - (1)\frac{e^{-st}}{s^2}\right]_0^3$$

$$= \frac{2}{3} \cdot \frac{1}{1-e^{-3s}} \left[\frac{3e^{-3s}}{-s} - \frac{e^{-3s}}{s^2} + \frac{1}{s^2}\right]$$

$$= \frac{2}{3} \frac{1}{1-e^{-3s}} \left[\frac{3e^{-3s}}{-s} + \frac{1-e^{-3s}}{s^2}\right]$$

$$= \frac{2e^{-3s}}{-s(1-e^{-3s})} + \frac{2}{3s^2}$$

Scanned by CamScanner

Find the Laplace transform of a square wave,

$f(t) = a$ for $0 < t < T/2$

$= 0$ for $T/2 < t < T$

and $f(t+T) = f(t)$ [Period = T]

Ans. $L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$

$= \frac{1}{1-e^{-sT}} \left[\int_0^{T/2} e^{-st} a\,dt + \int_{T/2}^{T} e^{-st} \cdot 0\,dt \right]$

$= \frac{1}{1-e^{-sT}} \left[a \cdot \frac{e^{-st}}{-s} \right]_0^{T/2}$

$= \frac{a}{1-e^{-sT}} \left[\frac{e^{-sT/2}}{-s} + \frac{1}{s} \right]$

$= \frac{a(1-e^{-sT/2})}{s(1-e^{-sT})}$

Q.3. Draw the graph of the periodic function

$$f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$$

and find its Laplace transform.

Soln:- Period $= 2\pi = T$

$$L\{f(t)\} = \frac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-sT}}$$

$= \frac{1}{1-e^{-2\pi s}} \int_0^{2\pi} e^{-st} f(t)\,dt$

$= \frac{1}{1-e^{-2\pi s}} \left[\int_0^{\pi} e^{-st} \cdot t\,dt + \int_{\pi}^{2\pi} e^{-st} (\pi - t)\,dt \right]$

$= \frac{1}{1-e^{-2\pi s}} \left[\left[\frac{t e^{-st}}{-s} - 1 \cdot \frac{e^{-st}}{(-s)^2} \right]_0^{\pi} + \left[(\pi - t) \frac{e^{-st}}{(-s)} - (-1) \frac{e^{-st}}{(-s)^2} \right]_{\pi}^{2\pi} \right]$

$= \frac{1}{1-e^{-2\pi s}} \left[\left[\frac{\pi e^{-\pi s}}{-s} - \frac{e^{-\pi s}}{(-s)^2} - 0 + \frac{1}{s^2} \right] \right]$

Scanned by CamScanner

$$+\left[(\lambda-2\lambda)\frac{e^{-2\lambda s}}{-s}+\frac{e^{-2\lambda s}}{s^2}-(\lambda-\lambda)\frac{e^{-\lambda s}}{-s}-\frac{e^{-\lambda s}}{s^2}\right]$$

$$=\frac{1}{1-e^{-2\lambda s}}\left\{-\frac{\lambda e^{-\lambda s}}{s}-\frac{e^{-\lambda s}}{s^2}+\frac{1}{s^2}+\frac{\lambda e^{-2\lambda s}}{s}+\frac{e^{-2\lambda s}}{s^2}-0-\frac{e^{-\lambda s}}{s^2}\right\}$$

$$=\frac{1}{1-e^{-2\lambda s}}\left\{\frac{-\lambda}{s}e^{-\lambda s}+\frac{\lambda}{s}e^{-2\lambda s}+\frac{1}{s^2}-\frac{1}{s^2}e^{-\lambda s}+\frac{1}{s^2}e^{-2\lambda s}-\frac{e^{-\lambda s}}{s^2}\right\}$$

$$=\frac{1}{1-e^{-2\lambda s}}\left[\frac{\lambda}{s}(e^{-2\lambda s}-e^{-\lambda s})+\frac{1}{s^2}(1+e^{-2\lambda s}-2e^{-\lambda s})\right]$$

$$=\frac{-\lambda s e^{-\lambda s}(1-e^{-\lambda s})+(1-e^{-\lambda s})^2}{s^2(1+e^{-\lambda s})(1-e^{-\lambda s})}$$

$$=\frac{-\lambda s e^{-\lambda s}+1-e^{-\lambda s}}{s^2(1+e^{-\lambda s})}$$

Q. Find the Laplace transform of the function (Half-wave rectifier)

$$f(t)=\begin{cases}\sin\omega t, & 0<t<\pi/\omega\\ 0, & \pi/\omega<t<2\pi/\omega\end{cases}$$

Ans) Period $= T = 2\pi/\omega$

$$L[f(t)]=\frac{1}{1-e^{-s(2\pi/\omega)}}\int_0^{2\pi/\omega}e^{-st}f(t)\,dt$$

$$=\frac{1}{1-e^{-2\pi s/\omega}}\left[\int_0^{\pi/\omega}e^{-st}\sin\omega t\,dt+\int_{\pi/\omega}^{2\pi/\omega}e^{-st}\cdot 0\,dt\right]$$

$$=\frac{1}{1-e^{-2\pi s/\omega}}\left[\frac{e^{-st}(-s\sin\omega t-\omega\cos\omega t)}{(-s)^2+\omega^2}\right]_0^{\pi/\omega}$$

$$=\frac{1}{1-e^{-2\pi s/\omega}}\left[\int e^{ax}\sin bx\,dx=\frac{e^{ax}(a\sin bx-b\cos bx)}{a^2+b^2}\right]$$

Here $a=-s$, $b=\omega$

Scanned by CamScanner

$$\Rightarrow L[f(t)] = \frac{1}{1-e^{-2\pi s/\omega}} \left[\frac{\{e^{-s\pi/\omega}(-s\sin\omega\cdot\pi/\omega + \omega\cos\omega\pi/\omega)\} - e^{0}(-s\sin 0 + \omega\cos 0)}{\omega^2+s^2} \right]$$

$$= \frac{1}{1-e^{-2\pi s/\omega}} \left[\frac{e^{-s\pi/\omega}(+\omega) + \omega}{\omega^2+s^2} \right]$$

$$= \frac{\omega(1+e^{-s\pi/\omega})}{(1+e^{-s\pi/\omega})(1-e^{-s\pi/\omega})(\omega^2+s^2)}$$

$$= \frac{\omega}{(\omega^2+s^2)(1-e^{-s\pi/\omega})}$$

Q. Find the Laplace transform of the triangular wave function of period $2c$ given by,

$$f(t) = \begin{cases} t, & 0<t<c \\ 2c-t, & c<t<2c \end{cases}$$

Q. $f(t) = \begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases}$

Q. $f(t) = \begin{cases} t, & 0<t<1/2 \\ 0, & 1/2<t<2a \end{cases}$

Q. $f(t) = \begin{cases} \cos\omega t, & 0<t<\pi/\omega \\ 0, & \pi/\omega<t<2\pi/\omega \end{cases}$ Ans: $\frac{s}{(s^2+\omega^2)(1-e^{-\pi s/\omega})}$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

Scanned by CamScanner

Unit Step Function

The unit step function $u(t-a)$ is defined as follows:

$$u(t-a) = \begin{cases} 0, & \text{when } t<a \\ 1, & \text{when } t \geq a \end{cases} \quad \text{where } a \geq 0$$

Laplace transform of unit step function

$$L[u(t-a)] = \frac{e^{-as}}{s}$$

$$\Rightarrow L[u(t-a)] = \int_0^\infty e^{-st}\, u(t-a)\, dt$$

$$= \int_0^a 0 \cdot e^{-st}\, dt + \int_a^\infty 1 \cdot e^{-st}\, dt$$

$$= \frac{e^{-st}}{-s}\Big|_a^\infty = \frac{e^{-as}}{s}$$

Q. $f(t) = \begin{cases} E & a<t<b \\ 0, & t \geq b \end{cases}$

$$= E \begin{cases} 1, & a<t<b \\ 0, & t \geq b \end{cases}$$

Ans) $L\{f(t)\} = E\left[\frac{e^{-as}}{s} - \frac{e^{-bs}}{s}\right]$

Q. $b(t) = \begin{cases} 8, & t<2 \\ 6, & t \geq 2 \end{cases}$

$$= \begin{cases} 8+0, & t<2 \\ 8-2, & t \geq 2 \end{cases} \quad = 8 + \begin{cases} 0, & t<2 \\ -2, & t \geq 2 \end{cases}$$

$$= 8 + (-2) \begin{cases} 0, & t<2 \\ 1, & t \geq 2 \end{cases}$$

$$= 8 - 2u(t-2)$$

$$L\,p(t) = 8L(1) - 2L\{u(t-2)\}$$

$$= 8/s - 2\frac{e^{-2s}}{s}$$

Scanned by CamScanner

Second shifting theorem

Q If $L[F(t)] = f(s)$ then $L[f(t-a)\cdot u(t-a)] = e^{-as} f(s)$

Q. Express the following function in terms of unit step function and find its Laplace transform.

$$f(t) = \begin{cases} t-1, & 1<t<2 \\ 3-t, & 2<t<3 \end{cases}$$

Ans)

$$f(t) = \begin{cases} t-1; & 1<t<2 \\ 3-t; & 2<t<3 \end{cases}$$

$$= (t-1)[u(t-1) - u(t-2)] + (3-t)[u(t-2) - u(t-3)]$$

$$= (t-1)u(t-1) - (t-1)u(t-2) + (3-t)u(t-2) - (3-t)u(t-3)$$

$$= (t-1)u(t-1) - u(t-2)[t-1-3+t] + (t-3)u(t-3)$$

$$= (t-1)u(t-1) - u(t-2)(2t-4) + (t-3)u(t-3)$$

$$= (t-1)u(t-1) - 2(t-2)u(t-2) + (t-3)u(t-3)$$

$$= e^{-1\cdot s} L(t) - 2e^{-2s} L(t) + e^{-3s} L(t)$$

$$= \frac{e^{-s}}{s^2} - \frac{2e^{-2s}}{s^2} + \frac{e^{-3s}}{s^2}$$

Q. $F(t) = \begin{cases} \sin(t-\pi/3), & t < \pi/3 \\ 0, & t < \pi/3 \end{cases}$

$$L\{F(t)\} = e^{-s\pi/3} L(\sin t)$$

$$= e^{-s\pi/3} \cdot \frac{1}{s^2+1}$$

Scanned by CamScanner

Theorem :- $L[f(t)u(t-a)] = e^{-as}[f(t+a)]$

Q $t^2 u(t-3)$

$L[f(t)] = L[t^2 u(t-3)]$

Q. Find the Laplace transform of $e^{-2t}u_\pi(t)$ where,

$$u_\pi(t) = \begin{cases} 0, & t<\pi \\ 1, & t>\pi \end{cases}$$

$u_\pi(t) = u(t-\pi)$

$$L[u_\pi(t)] = L[u(t-\pi)] = \frac{e^{-\pi s}}{s}$$

$$\Rightarrow L[e^{-2\pi}u_\pi(t)] = \frac{e^{-\pi(s+2)}}{s+2}$$

Q. $f(t) = \begin{cases} 0, & 0<t<1 \\ t-1, & 1<t<2 \\ 3, & 2<t \end{cases}$

$f(t) = 0$

$= (t-1)[u(t-1) - u(t-2)] + u(t-2)$

$= (t-1)u(t-1) - u(t-2)(t-1) + u(t-2)$

$= (t-1)u(t-1) - u(t-2)[t-1-1]$

$= (t-1)u(t-1) - (t-2)u(t-2)$

$L[f(t)] = L(t-1)u(t-1) - L(t-2)u(t-2)$

$$= \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s^2}$$

Q. $f(t) = \begin{cases} \sin 2t & 2\pi < t < 4\pi \\ 0 & \text{otherwise} \end{cases}$

$L[f(t)] = L[\sin 2t \cdot u(t-2\pi)] - L[\sin 2t \cdot u(t-4\pi)]$

$= e^{-2\pi s} L \sin 2(t+2\pi) - e^{-4\pi s} L \sin 2(t+4\pi)$

Scanned by CamScanner

$= e^{-2\pi s} L \sin 2t - e^{-4\pi s} L(\sin 2t)$

$= e^{-2\pi s} \frac{2}{s^2+4} - e^{-4\pi s} \frac{2}{s^2+4}$

$F(s) = (e^{-2\pi s} - e^{-4\pi s}) \frac{2}{s^2+4}$

Q. $f(t) = \begin{cases} t, & 0<t<2 \\ 0, & 2<t \end{cases}$ $\rightarrow \frac{1-(2s+1)e^{-2s}}{s^2}$

Q. $f(t) = \begin{cases} \sin t & 0<t<\pi \\ t & t>\pi \end{cases}$ $\rightarrow \frac{1+e^{-\pi s}}{s^2+1} + e^{-\pi s}$

Q. $f(t) = \begin{cases} 4 & 0<t<1 \\ -2 & 0<t<3 \\ 5 & t>3 \end{cases}$ $\rightarrow \frac{4-6e^{-s}+7e^{-3s}}{s}$

Q. $e^t u(t-c) = f(t)$ Q. $t^2 u(t-2) = f(t)$

Q. $L[t^2 u(t-3)] = e^{-3s} L(t+3)^2$

$= e^{-3s} L[t^2+6t+9]$

$= e^{-3s}[2/s^3 + 6/s^2 + 9/s]$

Q. $f(t) = \sin t\, u(t-4)$ $\rightarrow \frac{e^{-4s}}{s^2+1}[\cos 4 + s \sin 4]$

Scanned by CamScanner

Laplace transform of Dirac-delta function

The function $\delta(x)$ which is zero everywhere except at $x=0$ and tends to ∞ in such a manner that,

$$\int_{-\infty}^{\infty} \delta(x)\,dx = 1$$

$$\delta(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ \infty, & \text{if } x = 0 \end{cases}$$

Now one of the property of Dirac-delta function

$$\int_0^{\infty} f(t)\,\delta(t-a)\,dt = f(a)$$

$$\Rightarrow \int_0^{\infty} e^{-st}\,\delta(t-a)\,dt = f(a) = \left[e^{-st}\right]_{t=a} = e^{-sa}$$

$$\Rightarrow \boxed{L[\delta(t-a)] = e^{-sa}}$$

If $a=0$, then $L[\delta(t)] = 1$.

Ex - $L[t^3\delta(t-4)]$, $a=4$,

$$= \int_0^{\infty} e^{-st}\,t^3\,\delta(t-4)\,dt = 4^3 e^{-4s}$$

Ex - $L[e^{-4t}\delta(t-3)]$ $a=3$,

$$= \int_0^{\infty} e^{-st} e^{-4t}\,\delta(t-3)\,dt = e^{-12}\,e^{-3s} = e^{-3(s+4)}$$

Ex - $L\left[\frac{\delta(t-4)}{4}\right] = \frac{e^{-4s}}{4}$

Ex - $L[\cos t \log t\,\delta(t-\pi)] = \cos\pi \log\pi\, e^{-\pi s}$

$$= -\log\pi\, e^{-\pi s}.$$

Scanned by CamScanner

Fourier Transformation

The fourier integral transform $F(s)$ of a function $f(t)$ with the kernel $K(s,x)$ is defined as,

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{isx} f(x)\, dx$$

$$\Rightarrow f(x) = F^{-1}[F(s)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-isx}\, ds$$

List of Formulae for Fourier Integrals:-

1. Fourier integral for $f(x)$ is, $f(x) = \frac{1}{\pi} \int_0^{\infty} \int_{-\infty}^{\infty} f(t) \cos u(t-x)\, du\, dt$

2. Fourier sine integral for $f(x)$ is, $f(x) = \frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(t) \sin ut \sin ux\, du\, dt$

3. Fourier cosine integral for $f(x)$ is, $f(x) = \frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(t) \cos ut \cos ux\, du\, dt$

Fourier complex integrals :- $f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iux}\, du \int_{-\infty}^{\infty} f(t) e^{iut}\, dt$

Some useful integrals

1. $\int_0^{\infty} \frac{\sin ax}{x}\, dx = \frac{\pi}{2}$

2. $\int_0^{\infty} e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$

3. $\int_{-\infty}^{\infty} \frac{\sin mx}{(x-b)^2 + a^2}\, dx = \frac{\pi}{a} e^{-am} \sin bm$

Scanned by CamScanner